Theory – The lifetime of an option

We discovered what calls and puts (and combinations of these) are worth at expiry. Thanks to put-call-parity, we can even deduce the value of synthetics (as well as boxes and jelly rolls) at any time before expiry. Keep in mind, that this says *nothing* about the valuation of the individual calls and puts. For that, we need to take a closer look into the lifetime of an option, and the distribution of the stock (returns).

Imagine a 100-strike call option that you are long. If the stock trades below 100 at expiry, your call is worthless. But what about this call 3 months before expiry?

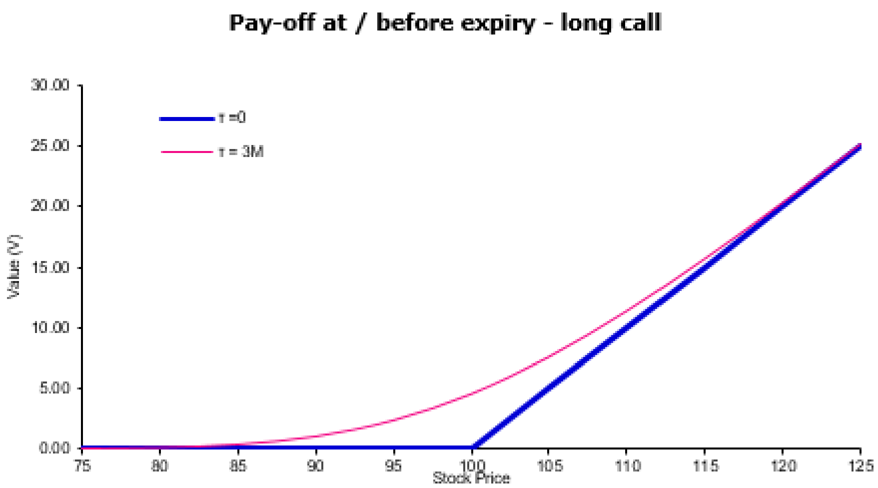

Figure 1: long the 100-call option

As you can see in Figure 1: if $S = 95$, the option is worthless at expiry. However, 3 months before expiry, the call option still has *some* value. This has to do with the fact that the stock might still move up through the strike price, such that the option lands in-the-money.

There are a few factors playing a role in how much this 100-call is exactly worth at each point in time. A few are obvious, some are less trivial:

- Time to maturity τ: clearly the more time, the higher the probability that the call might land in-the-money. So, the more time, the higher the value of the call;
- The stock price S: the higher the stock price, the higher the chance the call will land in-the-money;
- Implied volatility σ: this variable is essential throughout all option's theory and it is only the first time I mention it. Implied volatility is a measure of how volatile the stock is priced to move between now and expiry. The higher this number, the bigger the probability becomes that the call will land in-the-money;
- Dividends: sometimes a company decides to pay out part of their profits. On *ex-dividend day*, the stock will drop by the amount of the dividend. So a higher dividend implies a lower future price and it will affect the price of the call option;
- Interest rates r: to explain why interest rates affect the option prices, consider the case where $S = 120$. The pay-off of the option is then approximately the same to the pay-off of the stock itself (compare both graphs in figure 1 when $S = 120$). The only difference between the position in the call versus the position in the stock is the invested cash: if you would buy the stock, you pay 120, but if you would buy the 100-call, you pay only around 20. In a high interest rate environment, you would prefer the call over the stock (as you can invest the remaining 100 in a bank account).
- Typically, the strike price X is mentioned as well: a higher strike call option is worth less (as the stock needs to reach even higher to land in-the-money).

To summarize, the value of an option V (not just call options) is a function of the stock price, the strike price, the time to maturity, implied volatility, interest rates and dividends:

$$V := f(S, X, \tau, \sigma, r, D)$$

Intrinsic value versus time value

Have a look at Figure 1 again. If $S = 105$, the 3-months out option is already in-the-money, so worth at least $S - X = 105 - 100 = 5$. The quantity $S - X$ is called **intrinsic value** (for puts, $X - S$ is the intrinsic part).

For American style call options, it is easy to prove that $C \geq S - X$: if $C < S - X$, then you would buy the call for a price less than $S - X$ and immediately exercise it, yielding $S - X$. So if you see an American style option trading below intrinsic, buy as many as you can.

For European style call options, it works a bit different – as you need to hold your option position until expiry. Dividends and interest rates need to be considered.

Figure 1 shows that there is more value in the 100-call option if $S = 105$ than just intrinsic value. This part is called time value and is a function of volatility and interest rates. We can nicely illustrate this using PCP (for European options only – ignore dividends for now):

$$c - p = S - X + r \cdot X \cdot \tau \quad \Leftrightarrow$$

$$c = S - X + r \cdot X \cdot \tau + p$$

In this rearrangement of terms, you can clearly see the intrinsic value $S - X$ as part of the price of a European call. Note that $S - X$ can be negative and in that case the put is in in-the-money. The term $r \cdot X \cdot \tau$ is the component where interest rates affect the call price. The put value acts as *protection* – as we will see later, but for now it is sufficient to understand that the implied volatility σ is embedded in the put price (and hence the value of the put is part of the time value of the option, together with $r \cdot X \cdot \tau$).

Let's look at another example: for the sake of visualization, I took a more extreme example (a high volatility, $X = 40$, $r = 10\%$):

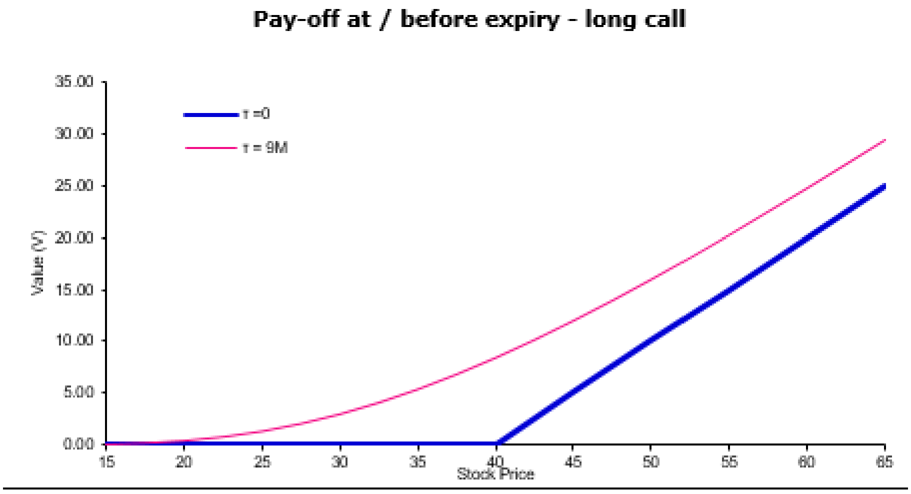

Figure 2: long the 40-call option

Assume the stock trades $S = 55$ and let's zoom in a bit:

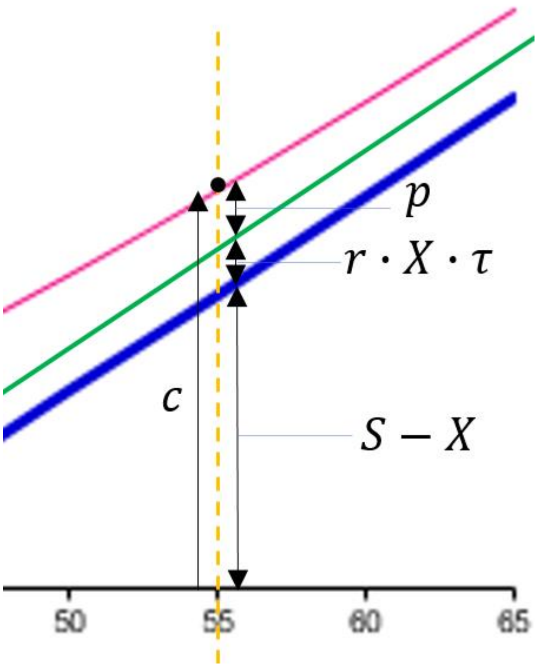

Figure 3: long the 40-call option, $S = 55$

In Figure 3, we observe the decomposition of the European call price. Also note that asymptotically, the call price converges to $S - X + r \cdot X \cdot \tau$ (and not just $S - X$) as S rises. This makes sense: if the stock moves to $S = 300$, the option will surely land in-the-money. The *protection*, the 40-strike put, is then worthless. However, you are still collecting interest during the lifetime of the option.

Also have a look at a second decomposition below:

$$c = S - (X - r \cdot X \cdot \tau) + p$$

This decomposition shows that we can construct a call by buying a stock and borrowing an amount of cash worth the discounted value of the strike price + buying the adjacent put option. This means we have a levered stock position (*on margin*) plus the downside protection.

American style options

We already warned that in general PCP does not hold for American style options. However, in the absence of dividends and with positive interest rates – we can easily prove that an American style call option C has to be worth the same as its European style equivalent c:

- $C \geq c$: all specifications are the same, but in case of an American style option you have an extra feature: your choice of exercising *before* maturity. Hence, by definition it must be worth at least c

- We showed before that $C \geq S - X$. So, at every point in time, it is better to sell the American style option, rather than exercising it early (in case you would only get $S - X$). Given that there is no incentive to exercise early, it follows that $C = c$.[1]

So for non-dividend paying stocks and $r > 0$, we can adopt PCP for American style call options[2]:

$$C = S - X + r \cdot X \cdot \tau + p$$

We call this the PCP "regrets" composition (Falcon Crack, 2014). By **early exercising** C, you will only receive $S - X$. You will not benefit from the interest rate you receive during the lifetime of the option. The reason for this is the fact that by early exercising, you spend X now, instead of at τ – foregoing the difference between the present value of X and X itself (precisely the term $r \cdot X \cdot \tau$). Next, you also give up your protection p by early exercising.

Given that both $r \cdot X \cdot \tau$ and p are positive, early exercising in the absence of dividends is never beneficial.

Lastly: be aware that the aforementioned calculations are more than just an academic exercise. It is really important that you are able to make the right decisions when pricing your calls and puts and especially when to (not) exercise them[3].

[1] Later, we will see that $C = S - X$ only at expiry (so equivalent to European style options) or in case there *is* a dividend and only on the evening before the stock goes ex-dividend
[2] American style put options might be exercised early as you will see in a different module
[3] Also a lesson taken from (Falcon Crack, 2014)